# Strict Self-Assembly of Discrete Sierpinski Triangles

Scott Summers
Iowa State University

Winfree (1998) showed that discrete Sierpinski triangles can self-assemble in the Tile Assembly Model. A striking molecular realization of this self-assembly, using DNA tiles a few nanometers long and verifying the results by atomic-force microscopy, was achieved by Rothemund, Papadakis, and Winfree (2004).

Precisely speaking, the above self-assemblies tile completely filled-in, two-dimensional regions of the plane, with labeled subsets of these tiles representing discrete Sierpinski triangles. This paper addresses the more challenging problem of the strict self-assembly of discrete Sierpinski triangles, i.e., the task of tiling a discrete Sierpinski triangle and nothing else.

We first prove that the standard discrete Sierpinski triangle cannot strictly self-assemble in the Tile Assembly Model. We then define the fibered Sierpinski triangle, a discrete Sierpinski triangle with the same fractal dimension as the standard one but with thin fibers that can carry data, and show that the fibered Sierpinski triangle strictly self-assembles in the Tile Assembly Model. In contrast with the simple XOR algorithm of the earlier, non-strict self-assemblies, our strict self-assembly algorithm makes extensive, recursive use of Winfree counters, coupled with measured delay and corner-turning operations. We verify our strict self-assembly using the local determinism method of Soloveichik and Winfree (2005).

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