Graphs with Rotation

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A rotation system for a graph specifies a clockwise ordering of the edges incident at each vertex of the graph. Rotation systems have traditionally been used to describe embeddings of graphs in surfaces. We have recently applied them to various problems concerned with graph drawings that contain crossings: odd crossing number, minor-monotone crossing number, the Hanani-Tutte theorem, and generalized thrackles.

In this talk we will survey some of the new applications of graphs with rotation systems and discuss the complexity of computing the crossing number of a graph with rotation system: We can show that computing the crossing number (or odd-crossing number or pair-crossing number) of a graph with rotation system is NP-hard. As a corollary we obtain Hlineny's result that computing the crossing number of a cubic graph (without rotation system) and computing the minor-monotone crossing number is NP-complete.

If we restrict the number of vertices to 1 or 2, the crossing number problem (with rotation) lies in P. The case of three vertices is open. For the pair crossing number even the case $k = 2$ is open.