HOMEWORK 3: DUE 15 OCT 2007

CSE 40833/60833: INTRO TO PARALLEL ALGORITHMS AND PROGRAMMING

Problem 3.1 (5 points)

Suppose n^2 processors were arranged in an n ? n mesh. Each processor can only communicate (directly) with its neighbors on the mesh. Initially, every processor gets an integer input. Design a parallel algorithm to sort these inputs on the mesh. Finally, all elements in a single row must be sorted in an increasing order from left to right, and all the rows must be sorted in increasing order (every element in the first row must be no larger than every element in the second row, and so on).

Use the following model: in each step of the parallel algorithm, every node can send a message to no more than one of its neighbors on the mesh, and can receive a message from no more than one of its neighbors on the mesh. Analyze the run time of the algorithm and its efficiency.

Problem 3.2 (5 points)

Draw the detailed bitonic sorting circuit for sorting 4 numbers. Use horizontal lines to denote the processors and vertical lines to denote comparators. Using the diagram, illustrate how the input 7, 2, 4, 10 is sorted. Just show the flow of numbers on the bitonic sorting circuit by indicating numbers on the lines whenever the numbers change due to comparators. No verbal explanation is required.

Problem 3.3 (5 points)

Give an algorithm to merge two sorted sequences of lengths m and n, respectively. You may assume that the input is an array of length $m + n$ with one sequence followed by the other, distributed across processors such that each processor has a subarray of size $\frac{m+n}{p}$. What are the computation and communication times for your algorithm?

Problem 3.4 (5 points)

Consider a tree of n nodes and having a bounded degree (i.e., the number of children per node is bounded a constant). The tree is stored in an array A of size n. Each node also contains the indices at which its parent and children are stored in the array. The array is distributed among processors in the usual way such that each processor is responsible for $\frac{n}{p}$ entries.

For each of the following operations, Which communication primitive will you use and what is the runtime? Assume message sizes are constant.

(a) Each node in the tree sends a message to its parent.
(b) Each node in the tree sends a message to all its children.