

CSE 411 - Lecture 2

Finite and Infinite Sets

Proof Techniques

- ⑥ **Partial Order** $R \subseteq A \times A$: reflexive, antisymmetric and transitive
 - △ **Minimal element** a : $(b, a) \in R$ iff $a = b$
 - △ Finite partial orders have at least one minimal element
 - △ Infinite partial orders may not have a minimal element
 - △ *Examples?*
- ⑥ **Total order**: partial order s.t.
 - △ $\forall a, b \in A$, either $(a, b) \in R$ or $(b, a) \in R$
 - △ *Example?*

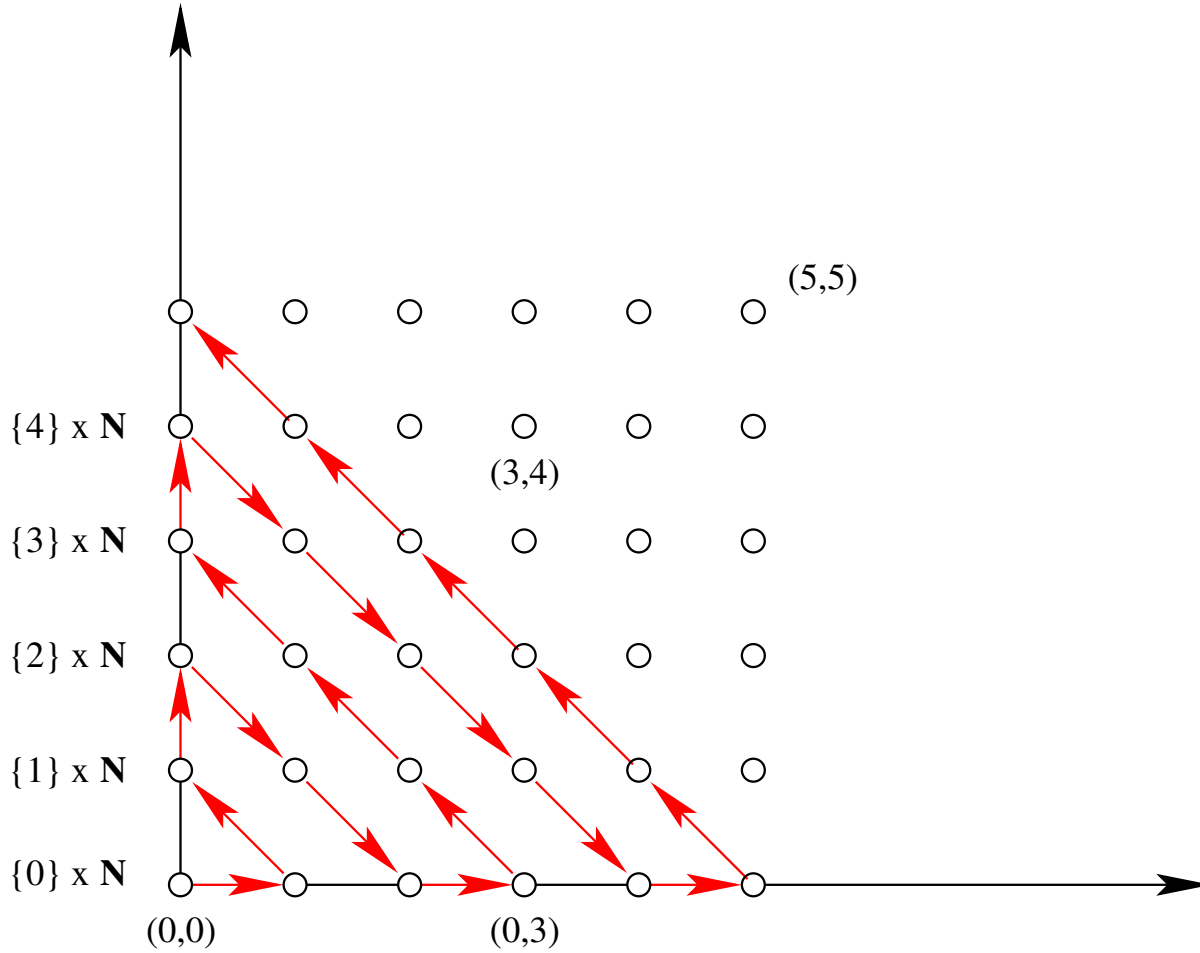
Finite and Infinite Sets

- ⑥ $|A|$: size of set A
- ⑥ **Equinumerous sets**: sets of same size
 - △ **Defn**: A and B are equinumerous iff there exists a bijection $f : A \mapsto B$
- ⑥ **Finite set** A : there exists a bijection between A and T_n , for some $n \in \mathbb{N}$
 - △ $T_n = \{1, 2, \dots, n\}$, $T_0 = \emptyset$
- ⑥ **Infinite set**: not finite

Countable and Uncountable Sets

- ⑥ Countably infinite set:
 - △ *Defn*: A is countably infinite if it is equinumerous with \mathbb{N}
- ⑥ Countable set: finite or countably infinite
- ⑥ Uncountable set: not countable
- ⑥ Finite unions of countable sets are countable

$\mathbb{N} \times \mathbb{N}$ is Countable



Proof Techniques

- ① Mathematical Induction
- ② The Pigeonhole Principle
- ③ The Diagonalization Principle

Mathematical Induction

- ⑥ **Theorem:** Let A be a set of natural numbers such that (1) $0 \in A$ and (2) $\forall n$, if $\{0, 1, 2, \dots, n\} \subseteq A$, then $n + 1 \in A$. Then $A = \mathbb{N}$.
- ⑥ Applying the induction technique: $A = \{n : P \text{ is true for } n\}$
 - ① **basis step:** show that $0 \in A$
 - ② **induction hypothesis:** assume that for some fixed $n \geq 0$ proposition P holds
 - ③ **induction step:** show that P is true for $n + 1$ (using induction hypothesis)

The Pigeonhole Principle

- ⑥ **Theorem:** If A and B are two finite sets with $|A| > |B|$, then there is no one-to-one function from A to B .
- ⑥ Example of application of the pigeonhole principle:
 - △ **Theorem:** Let R be a binary relation on a finite set A , and let $a, b \in A$. If there is a path from a to b in R , then there is path of length at most $|A|$

The Diagonalization Principle

- ⑥ **Theorem:** Let R be a binary relation on a set A , and let $D = \{a \in A : (a, a) \notin R\}$.
For each $a \in A$ let $R_a = \{b \in A : (a, b) \in R\}$.
Then D is distinct from each R_a .

Example

$$R = \{ (a, b), (a, d), (b, b), (b, c), (c, c), (d, b), (d, c), (d, e), (d, f), (e, e), (e, f), (f, a), (f, c), (f, d), (f, e) \}$$

	a	b	c	d	e	f
a		×		×		
b		×	×			
c			×			
d		×	×		×	×
e					×	×
f	×		×	×	×	

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