

Finite state transducers

Regular finite state machines are used as language recognizers. But it is a simple matter to augment the model to allow for output at each step of a machine's operation. In such situations often we may no longer care whether the machine accepts any strings. Many finite state transducers are loops that run forever, processing inputs.

We'll look at different types of finite state transducers. The first kind associates an output with each state of a machine M . That output is generated whenever M enters the associated state. Deterministic finite state transducers of this sort are called Moore machines (after their inventor).

Definition 1 A Moore machine M is a seven-tuple $(Q, \Sigma_{in}, \Sigma_{out}, \delta, D, q_0, F)$, where:

- Q is a finite set of states,
- Σ_{in} is a finite input alphabet,
- Σ_{out} is a finite output alphabet,
- $q_0 \in Q$ is the start state,
- $F \subseteq Q$ is the set of accepting states (this designation might be optional for some applications),
- $\delta : Q \times \Sigma_{in} \rightarrow Q$ is the transition function,
- $D : Q \rightarrow \Sigma_{out}^*$ is the display function.

A Moore machine M computes a function $f(w)$ if and only if, when it reads the input string w , its output sequence is $f(w)$. An example of Moore machines is a controller for a traffic light.

A different type of a finite state transducer permits each machine to output any finite sequence of symbols as it makes each transition (i.e., as it reads each symbol of its input). DFAs of this kind are called Mealy machines.

Definition 2 A Mealy machine M is a six-tuple $(Q, \Sigma_{in}, \Sigma_{out}, \delta, q_0, F)$, where:

- Q is a finite set of states,
- Σ_{in} is a finite input alphabet,
- Σ_{out} is a finite output alphabet,
- $q_0 \in Q$ is the start state,
- $F \subseteq Q$ is the set of accepting states,
- $\delta : Q \times \Sigma_{in} \rightarrow Q \times \Sigma_{out}^*$ is the transition function.

A Mealy machine M computes a function $f(w)$ if and only if, when it reads the input string w , its output sequence is $f(w)$. An example of Mealy machines is a circuit that adds parity bits to digital communication.